VIVEK TUTORIALS

Preliminary Examination [MODEL ANSWER]

Std: SSC (E.M) **Subject: Mathematics II** **Time: 2 Hours**

Date : 23/Jan/2020 **Max Marks: 40**

(i) All questions are compulsory.
(ii) Use of calculator is not allowed.
(iii) Total marks are shown on the right side of the question.

Q.1 (A) Choose the correct alternative: **4**

(1) Ans. (b)

$\Delta DEF \sim \Delta ABC$

AB = 3 cm, BC = 2 cm, CA = 2.5 cm, EF = 4 cm

$\because$ Δs are similar

$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA}$$

$$\Rightarrow \frac{DE}{3} = \frac{4}{2} = \frac{FD}{2.5}$$

$$\text{Now } \frac{DE}{3} = \frac{4}{2}$$

$$\Rightarrow DE \ \frac{3 \times 4}{2} = 6 \text{ cm}$$

$$\text{and FD} = \frac{4}{2} \Rightarrow FD = \frac{4 \times 2.5}{2} = 5 \text{ cm}$$

$\therefore$ Perimeter of ΔDEF

= 6 + 4 + 5 = 15 cm (b)

(2) Ans. (a) parallel

(3) Ans. (c)

Area of sector of a circle = $\frac{5}{18}$ × area of circle

Let θ be its angle at the centre and r be radius

$$\text{Then, } \pi r^2 \times \frac{\theta}{360^\circ} = \frac{5}{18}\pi r^2$$

$$\frac{\theta}{360^\circ} = \frac{5}{18} \Rightarrow \theta = \frac{5}{18} \times 360^\circ = 100^c \quad (c)$$

(4) Ans. (C) 5

(B) Solve the following: **4**

(1) Ans. $\frac{A(\Delta ABC)}{A(\Delta DBC)} = \frac{AE}{DF}$ .... bases are equal, hence areas proportional to heights.

$= \frac{4}{6} = \frac{2}{3}$

(2) Ans. Write the proof with the help of the following steps.

(1) Draw ray RS. It intersects the circle at T.

(2) Show that RS = TS.

(3) Write a result using theorem of intersection of chords inside the circle.

(4) Using RS = TS complete the proof.

(3) Ans. $LHS = \tan^4\theta + \tan^2\theta$

$= \tan^2\theta(1 + \tan^2\theta)$

$= (\sec^2\theta - 1)(\sec^2\theta)$

$\left[\begin{array}{l} \because 1 + \tan^2 A = \sec^2 A \\ \therefore \tan^2 A = \sec^2 A - 1 \end{array}\right]$

$LHS = \sec^4\theta - \sec^2\theta$

$RHS = \sec^4\theta - \sec^2\theta$

LHS = RHS.

$\therefore \tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta.$

(4) Ans. $T(-3, 6) = (x_1, y_1)$

$R(9, -10) = (x_2, y_2)$

By distance formula,

$d(T, R) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[9 - (-3)]^2 + (-10 - 6)^2}$

$= \sqrt{(9 + 3)^2 + (-16)^2}$

$= \sqrt{(12)^2 + 256}$

$= \sqrt{144 + 256}$

$= \sqrt{400}$

$\therefore d(T, R) = 20$ units

Q.2 (A) Complete the following activities:(Any TWO) **4**

(1) Ans. P(x, y) divides seg AB in the ratio 2 : 3.

$A(-1, 7) = (x_1, y_1)$

$B(4, -3) = (x_2, y_2)$

m : n = 2 : 3

By section formula,

$x = \frac{mx_2 + nx_1}{m + n}$; and $y = \frac{my_2 + ny_1}{m + n}$

$= \frac{2 \times 4 + 3 \times (-1)}{2 + 3}$ and $\frac{2 \times \boxed{(-3)} + 3 \times (7)}{2 + \boxed{3}}$

$= \frac{8 - 3}{5}$ and $\frac{-6 + \boxed{21}}{5}$

$= \frac{5}{5}$ and $\boxed{\frac{15}{5}}$

x = 1 and y = 3

$\therefore$ The coordinates of point P are (1, 3).

(2) Ans. For a sector, $r = 18$ cm, $\theta = 80$

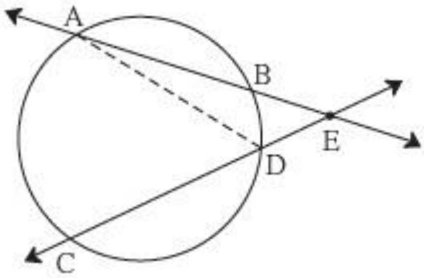
Length of an arc $= \frac{\boxed{\theta}}{\boxed{360}} \times 2\pi r$

$= \frac{\boxed{80}}{\boxed{360}} \times 2 \times 3.14 \times \boxed{18}$

$= 3.14 \times 8$

$= \boxed{25.12 \text{ cm}}$

(3) Ans.



Given : Chords AB and CD intersect at $\boxed{E}$ in the $\boxed{\text{exterior}}$ of the circle.

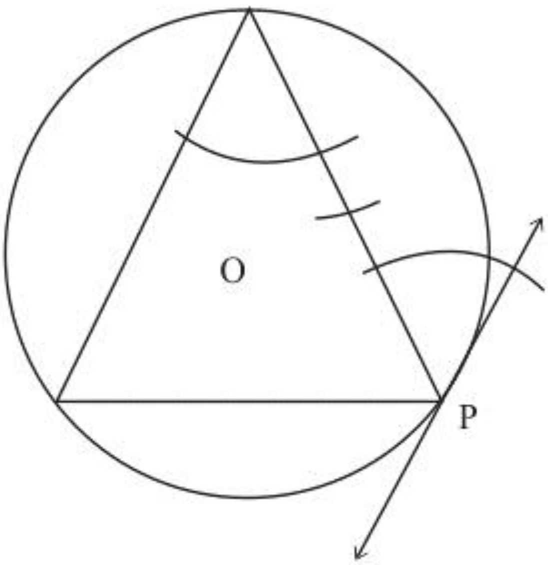
To prove: $\angle\boxed{AEC} = \frac{1}{2}[m(\text{arc } AC) - m(\boxed{\text{arc } BD})]$

Construction: Draw seg AD.

Consider angles of Δ AED and its exterior angle and write the proof.

(B) Solve the following: (Any FOUR) **8**

(1) Ans.



(2) Ans. Take a point R on line PQ such that P – Q – R.
$\angle APT = \angle BQR = 90^\circ$
.......1 [AP ⊥ PQ, BQ ⊥ PQ, given]
$\angle CTQ = 90^\circ$
........2 [Tangent and radius are ⊥ to each other at the point of contact]
$\therefore \angle APT \cong \angle BQR \cong \angle CTQ$ [From 1 & 2]
$\therefore AP \parallel CT \parallel BQ$
.... [Corresponding angles test]
$\therefore \frac{PT}{QT} = \frac{AC}{BC}$
.........3 [Property of intercepts made by three parallel lines]
AC = BC [Radii of a circle]
$\therefore \frac{AC}{BC} = 1$ 4
$\therefore \frac{PT}{QT} = 1$ [From 3 & 4]
$\therefore$ PT = QT 5
In ΔCTP and ΔCTQ
(i) side CT ≅ side CT [Common side]
(ii) $\angle CTP \cong \angle CTQ$ [Each 90°, from 2]
(iii) side PT ≅ side QT [From 5]
$\therefore \Delta CTP \cong \Delta CTQ$ [SAS Test]
$\therefore$ seg CP ≅ seg CQ [c.s.c.t]

(3) Ans. For the circle, r = 3.4 cm perimeter of sector P – ABC = 12.8 cm
P(P – ABC) = Length of arc (l) + r + r
$\therefore 12.8 = l + 3.4 + 3.4$
$\therefore 12.8 - 6.8 = l$
$\therefore l = 6$ cm
Area of the sector $= l \times \frac{r}{2}$
$= 6 \times \frac{3.4}{2} \times 7 \times 7$
Area of the sector = 10.2 cm^2
Area of the sector is 10.2 cm^2

(4) Ans. For the metallic cuboid,
l = 16 cm, b = 11 cm, h = 10 cm
For the cylindrical coin,
Diameter = 2 cm, Thickness (h_1) = 2mm = 0.2 cm
i.e. Radius (r_1) = 1 cm
Let number of coin made be N.
$\therefore$ N x volume of coin = Volume of cuboid
$\therefore N \times \pi r_1^2 h_1 = l \times b \times h$
$\therefore N \times \frac{22}{7} \times 1 \times 1 \times \frac{2}{10} = 16 \times 11 \times 10$
$\therefore N = \frac{16 \times 11 \times 10 \times 10 \times 7}{22 \times 2}$
$\therefore N = 2800$
$\therefore$ Number of coins made are 2800

(5) Ans. In ΔABC, seg BD bisects $\angle ABC$ [Given]

$\therefore \frac{AB}{BC} = \frac{AD}{DC}$

[Angle bisector property of a triangle]

$\therefore \frac{x}{x+5} = \frac{x-2}{x+2}$

$\therefore x(x+2) = (x+5)(x-2)$

$\therefore x^2 + 2x = x^2 + 5x - 2x - 10$

$\therefore x^2 + 2x = x^2 + 3x - 10$

$\therefore x^2 + 2x - x^2 - 3x = -10$

$\therefore -x = -10$

$\therefore x = 10$

Q.3 (A) Complete the following activity:(Any ONE) **3**

(1) Ans. □ PQRS is a cyclic quadrilateral [Given]

$\therefore \angle PQR + \angle PSR = 180°$

[Opposite angles of cyclic quadrilateral are supplementary]

$\therefore \angle PQR + 110° = 180°$

$\therefore \angle PQR = 180° - 110° = \boxed{70°}$ 1

$\angle PSR = \frac{1}{2} m$ (arc PQR)

[Inscribed angle theorem]

$\therefore 110° = \frac{1}{2} m$ (arc PQR)

$\therefore m$ (arc PQR) $= \boxed{220°}$ 2

In ΔPQR, side PQ $\cong$ side RQ [Given]

$\therefore \angle PQR \cong \angle QPR$

......3 [Isosceles triangle theorem]

In ΔPQR,

$\angle PQR + \angle PRQ + \angle QPR = 180°$

[Sum of all angles of a triangle is 180°]

$\therefore 70° + \angle QPR + \angle QPR = 180°$ [From 1 & 2]

$\therefore 2\angle QPR = 180° - 70°$

$\therefore 2\angle QPR = \boxed{110°}$

$\therefore \angle QPR = \boxed{55°}$ 4

$\angle QPR = \frac{1}{2} m$ (arc QR)

[Inscribed angle theorem]

$\therefore 55° = \frac{1}{2} \times m$ (arc QR) [From 3]

$\therefore m$ (arc QR) $= 110°$

$\angle PRQ = \boxed{55°}$ [From 3 & 4]

(2) Ans. $A(-4, 4) = (x_1, y_1)$

$K(-2, \frac{5}{2}) = (x_2, y_2)$

$N(4, -2) = (x_3, y_3)$

$$\text{Slope of line AK} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\frac{5}{2} - 4}{-2 - (-4)}$$

$$= \left(\frac{\boxed{5-8}}{2}\right) \div (\boxed{-2+4})$$

$$= \frac{-3}{2} \div 2$$

$$= \frac{-3}{2} \times \boxed{\frac{1}{2}}$$

$$\therefore \text{Slope of line AK} = \boxed{\frac{-3}{4}} \quad .....(i)$$

$$\text{Slope of line AN} = \frac{y_3 - y_1}{x_3 - x_1}$$

$$= \frac{-2-4}{4-(-4)}$$

$$= \frac{-6}{4+4}$$

$$= \boxed{\frac{-6}{8}}$$

$$\therefore \text{Slope of line AN} = \frac{-3}{4} \quad .....(ii)$$

$\therefore$ Slope of line AK = Slope of line $\boxed{AN}$

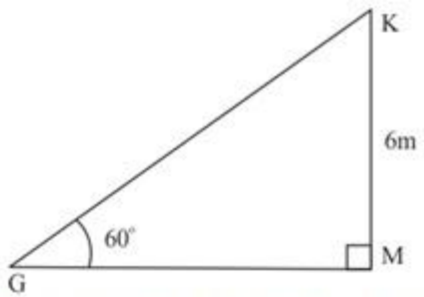
......[From (i) and (ii)]

Also, they have a common point A.

$\therefore$ Point A, K and N are collinear points.

(B) Solve the following: (Any TWO) **6**

(1) Ans.



'K' is position of height in the sky, 6m above the ground level i.e. KM = 6 m

$\angle KGM = 60°$

[Angle between thread and ground]

KG = Length of thread = ?

In ΔKMG, $\angle KMG = 90°$

$\therefore \sin \angle KGM = \frac{KM}{GK}$ [Definition]

$\therefore \sin 60° = \frac{6}{GK}$

$\therefore \frac{\sqrt{3}}{2} = \frac{6}{GK}$

$\therefore GK = \frac{6 \times 2}{\sqrt{3}} = \frac{12}{\sqrt{3}}$

$\therefore GK = \frac{12 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$

$\therefore GK = 4 \times 1.73 = 6.92$ m

$\therefore$ Length of the thread is 6.92 m

(2) Ans. : $\square$ ABCD is a parallelogram.

$\therefore$ AD || BC and AB || DC

Consider Δ ABC and Δ BDC.

Both the triangles are drawn in two parallel lines. Hence the distance between the two parallel lines is the height of both triangles.

In Δ ABC and Δ BDC, common base is BC and heights are equal.

Hence, $A(\Delta ABC) = A(\Delta BDC)$

In Δ ABC and Δ ABD, AB is common base and and heights are equal.

$\therefore A(\Delta ABC) = A(\Delta ABD)$

(3) Ans. Radius of a sphere, r = 30 cm

Radius of the cylinder, R = 10 cm

Height of the cylinder, H = 6 cm

Let the number of cylinders be n.

Volume of the sphere = n × volume of a cylinder

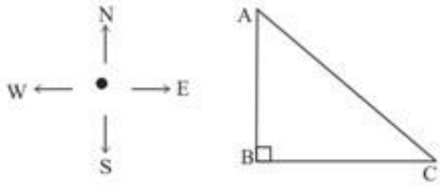
$$\therefore \; n = \frac{\text{Volume of the sphere}}{\text{Volume of a cylinder}}$$

$$= \frac{\frac{4}{3}\pi(r)^3}{\pi(R)^2 H}$$

$$= \frac{\frac{4}{3}\times(30)^3}{10^2\times 6} = \frac{\frac{4}{3}\times 30\times 30\times 30}{10\times 10\times 6} = 60$$

∴ 60 cylinders can be made .

(4) Ans.



B represent starting point of journey.

BA is distance travelled by Prasad in north direction

BC is distance travelled by Pranali in east direction

AC is distance between Pranali & Prasad after two hours

Let the speed of each one be x km/hr.

∴ Distance travelled by each one is 2x km/hr.

i.e. AB = BC = 2x km.

In ΔABC, $\angle B = 90°$

[Lines joining adjacent direction are ⊥ to each other]

$\therefore AB^2 + BC^2 = AC^2$ [Pythagoras theorem]

$\therefore (2x)^2 + (2x)^2 = (15\sqrt{2})^2$

$\therefore 4x^2 + 4x^2 = 225 \times 2$

$\therefore 8x^2 = 225 \times 2$

$x^2 = \frac{225 \times 2}{8} = \frac{225}{4}$

$\therefore x = \frac{15}{2}$ [taking square roots]

$\therefore x = 7.5$

Ans. speed of each one is 7.5 km/hr.

Q.4 Solve the following: (Any TWO) **8**

(1) Ans. In ΔPQR

$\angle PQR = 90°$

$\therefore PQ^2 + QR^2 = PR^2$ ---(i)[by Pythagoras theorem]

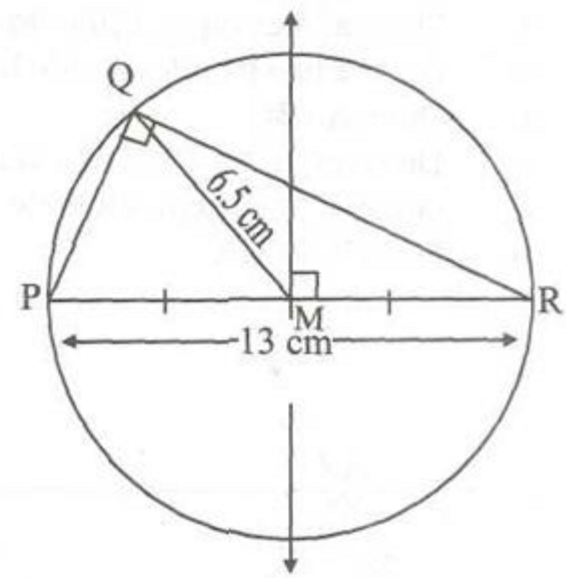
Since, $PQ^2 + QR^2 = 169$ ---(ii)[Given]

$\therefore PR^2 = 169$ ---[From (i) and (ii)]

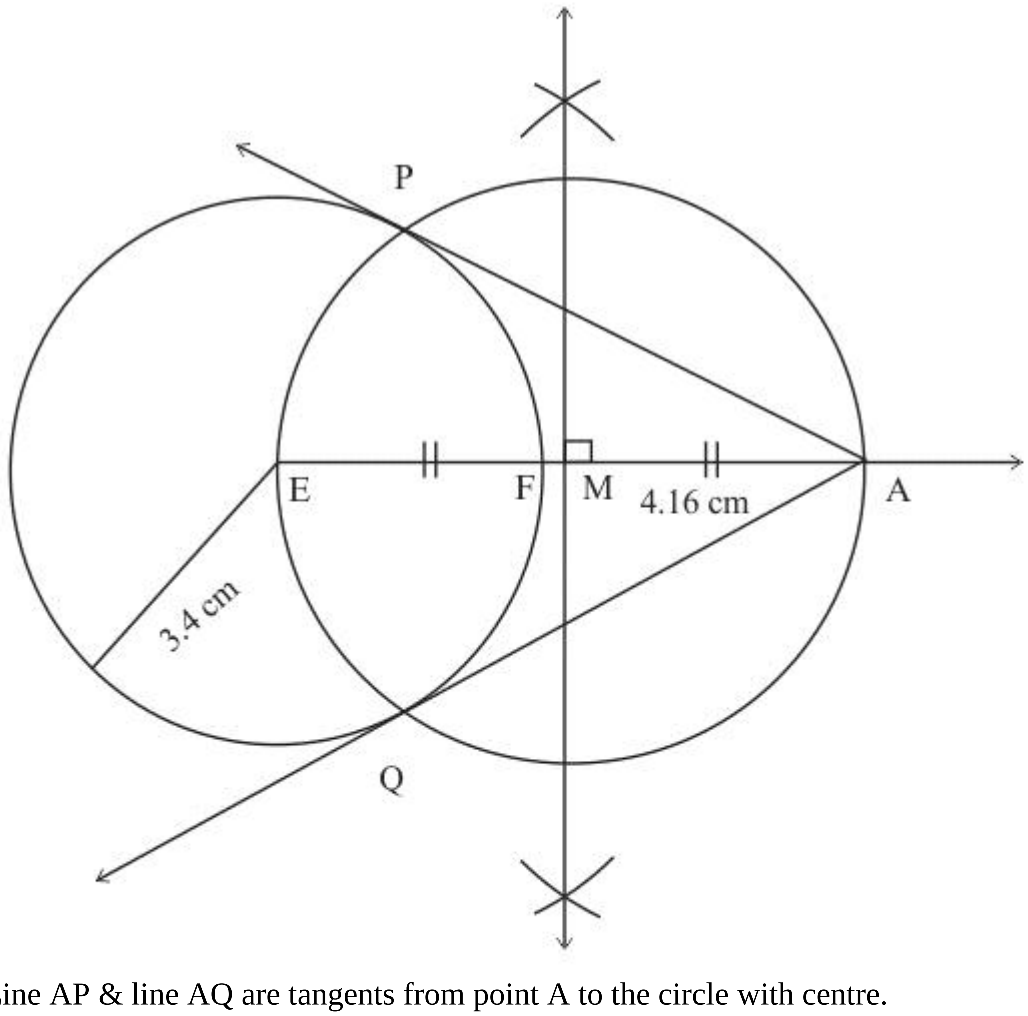
$\therefore PR = 13$ cm ---[Taking square root on both sides]

Steps of construction:

i. Draw seg PR = 13 cm.
Ii. Draw perpendicular bisector of PR. Let M be the midpoint.
Iii. With M as centre and radius PM = 6.5 cm, draw a circle passing through point P and R.
Iv. Take any point Q on the circle. Join PQ and QR.
V. ΔPQR is the required triangle.
Vi. The circle with centre M and radius MP is the required circumcircle of ΔPQR.



(2) Ans.



Line AP & line AQ are tangents from point A to the circle with centre.

(3) Ans. In ΔPRQ, $\angle PRQ = 90^\circ$ [Given]

$\therefore PQ^2 = PR^2 + QR^2$

[Pythagoras theorem] 1

$\therefore QR = 2\,RM$ ......2 [M midpoint of seg QR]

$\therefore PQ^2 = PR^2 + (2RM)^2$ [From 1 & 2]

$\therefore PQ^2 = PR^2 + 4RM^2$ 3

In ΔPRM, $\angle PRM = 90^\circ$ [Given]

$\therefore PM^2 = PR^2 + RM^2$ [Pythagoras theorem]

$\therefore RM^2 = PM^2 - PR^2$ 4

$\therefore PQ^2 = PR^2 + 4(PM^2 - PR^2)$ [From 3 & 4]

$\therefore PQ^2 = PR^2 + 4\,PM^2 - 4\,PR^2$

$\therefore PQ^2 = 4PM^2 - 3PR^2$

Q.5 Solve the following: (Any ONE) **3**

(1) Ans. Since CE || DA (construction)

$\angle CAD = \angle ACE$ (Alternate angles) ... (1)

$\angle DAK = \angle AEC$ (Corresponding angles) ... (2)

But $\angle CAD = \angle DAK$ (AD is the bisector of $\angle CAK$)... (3)

$\therefore \angle ACE = \angle AEC$ [From (1), (2) and (3)]

In ΔACE, we have

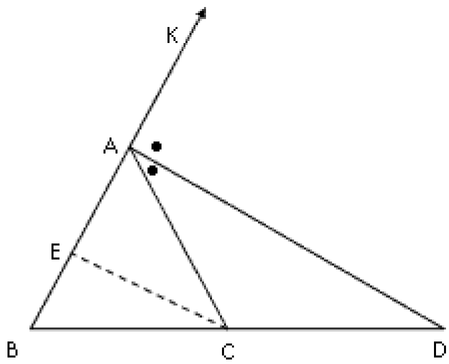
$\therefore AE = AC$ (Converse of isosceles Δ theorem)

$\frac{BE}{EA} = \frac{BC}{CD}$ (B. P.T.)

$\frac{BE+EA}{EA} = \frac{BC+CD}{CD}$ (Componendo)

$\frac{AB}{EA} = \frac{BD}{CD}$

$\frac{AB}{AC} = \frac{BD}{CD}$ [From (4)]



(2) Ans. L.H.S. $= (1 + \tan\theta)^2 + (1 + \cot\theta)^2$

$= 1 + 2\tan\theta + \tan^2\theta + 1 + 2\cot\theta + \cot^2\theta$

$= (1 + \tan^2\theta) + (1 + \cot^2\theta) + 2\tan\theta + 2\cot\theta$

$= \sec^2\theta + \text{cosec}^2\theta + \frac{2\sin\theta}{\cos\theta} + \frac{2\cos\theta}{\sin\theta}$

$= \sec^2\theta + \text{cosec}^2\theta + \frac{2\sin^2\theta + 2\cos^2\theta}{\cos\theta\sin\theta}$

$= \sec^2\theta + \text{cosec}^2\theta + \frac{2(\sin^2\theta + \cos^2\theta)}{\cos\theta\sin\theta}$

$= \sec^2\theta + \text{cosec}^2\theta + \frac{2(1)}{\sin\theta\cos\theta}$

$= \sec^2\theta + \text{cosec}^2\theta + 2\left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right)$

$= \sec^2\theta + \text{cosec}^2\theta + 2\sec\theta\,\text{cosec}\,\theta$

$= (\sec\theta + \text{cosec}\,\theta)^2$ [$\because (a + b)^2 = a^2 + b^2 + 2ab$]

= R.H.S.